Operations and Systems of Numbers (K-5)

revised 11/23/09

	COUNTING NUMBERS *1, 2, 3, … n*	**WHOLE NUMBERS** counting numbers ***n*** and ***0*** *0, 1, 2, 3 … m*	**FRACTIONS** whole numbers ***m***, ***n*** (***n*** ≠ *0*) and their divisions $\frac{0}{n}, \frac{1}{n}, \frac{2}{n}, \frac{3}{n}, \ldots \frac{m}{n}$	**RATIONAL NUMBERS** (positive) fractions $\frac{m}{n}$, their reflections $\frac{-m}{n}$ across *0* (negative fractions) $\frac{-m}{n} \ldots \frac{-3}{n}, \frac{-2}{n}, \frac{-1}{n}, \frac{0}{n}, \frac{1}{n}, \frac{2}{n}, \frac{3}{n}, \ldots \frac{m}{n}$
ADDITION $a+b=c$ ***b*** added onto ***a*** gives you ***c*** ***a*** and ***b*** can be **any two numbers**	***c* > *a*** [sums will ***always*** be greater]	if ***b* = *0***, then ***c* = *a*** *0 is the identity element.* if ***b* ≠ *0***, then ***c* > *a***, [sums will be greater]	if ***b* = *0***, then ***c* = *a*** *0 is the identity element.* if ***b* ≠ *0***, then ***c* > *a***, [sums will be greater]	if ***b* = *0***, then ***c* = *a*** *0 is the identity element.* if ***b* ≠ *0***, if ***b* > *0***, then ***c* > *a*** [sums will be greater] if ***b* < *0***, then ***c* < *a*** [sums will be less] if ***b* = *−a***, then ***c* = *0*** sums will be *0*. [*additive inverse*]
MULTIPLICATION $a \cdot b = d$ ***a*** copies of ***b*** gives you ***d*** ***a*** and ***b*** can be **any two numbers**	if ***a* > *1***, then ***d* > *b*** if ***a* = *1***, then ***d* = *b*** *1 is the identity element.*	if ***a* = *0***, then ***d* = *0*** if ***a* > *1***, then ***d* > *b*** if ***a* = *1***, then ***d* = *b*** *1 is the identity element*	if ***a* = *0***, then ***d* = *0*** if ***a* > *1***, then ***d* > *b*** if ***a* < *1***, then ***d* < *b*** if ***a* = *1***, then ***d* = *b*** *1 is the identity element.* if $a = \frac{1}{b}$ then ***d* = *1*** [*multiplicative inverse*]	if ***a* = *0***, then ***d* = *0*** ***sign*** *[positive or negative]:* if ***a* > *0***, then ***d*** and ***b*** have the same sign if ***a* < *0***, then ***d*** and ***b*** have opposite signs ***absolute value*** *[distance from 0]:* if ***|a|* > *1***, ***|d|* > *|b|*** products will be *farther from 0* if ***|a|* < *1***, ***|d|* < *|b|*** products will be *closer to 0* if ***|a|* = *1***, then ***|d|* = *|b|*** *1 is the identity element.* if $\|a\| = \frac{1}{\|b\|}$ then ***|d|* = *1*** [*multiplicative inverse*]
SUBTRACTION (from addition) $c-a=b$ means ***b*** is the number so that $a+b=c$	*Make sure* ***c* > *a***, then ***b*** will be the number added onto ***a*** that gives you ***c***			*For* ***any numbers***, ***a***, ***b***, *and* ***c***, ***b*** will be the number added onto ***a*** that gives you ***c***
DIVISION (from multiplication) $\frac{d}{a} = b$ means ***b*** is the number so that $a \cdot b = d$	*Make sure* ***d*** *is a multiple of* ***a***. Then ***b*** will be the number such that ***a*** copies of ***b*** gives you ***d***.	*Make sure* ***d*** *is a multiple of* ***a*** ***AND*** *that* ***a* ≠ *0***. Then ***b*** will be the number such that ***a*** copies of ***b*** gives you ***d***.	*Make sure* ***a* ≠ *0***. *Then* ***d***, ***a***, *and* ***b*** *can be* ***any numbers—including fractions***. (It is not necessary that ***d*** be an integer multiple of ***a***.) ***b*** will still be the number such that ***a*** copies of ***b*** gives you ***d***. *(Since* ***a***, ***b***, *and* ***d*** *can all be fractions, "****a*** *copies of* ***b*** *gives you* ***d****" has to be appropriately interpreted for fractions.)*	