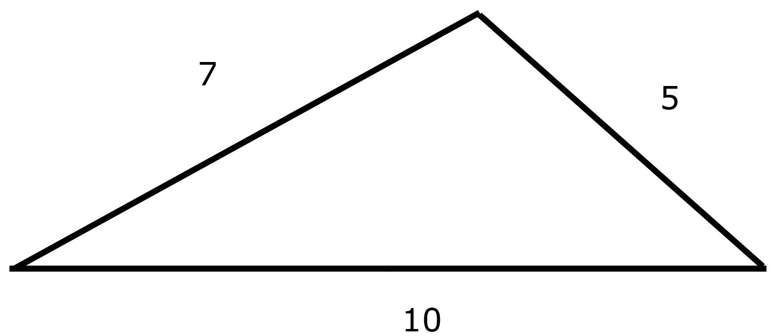
When triangles have relationships…

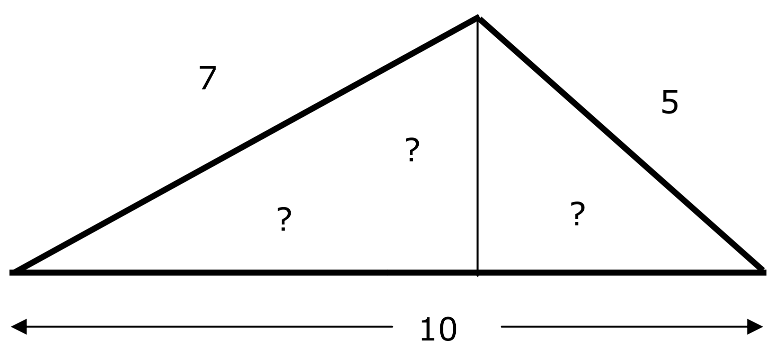
(Finding the area of any old triangle if you know the sides)

Let's start with a generic-looking triangle:



We could find the area if we knew the height—and maybe we could find the height if we knew the parts of the big base that make up the little right triangles with the height.

But we don't know any of that stuff, so let's just put question marks for the things we don't know:

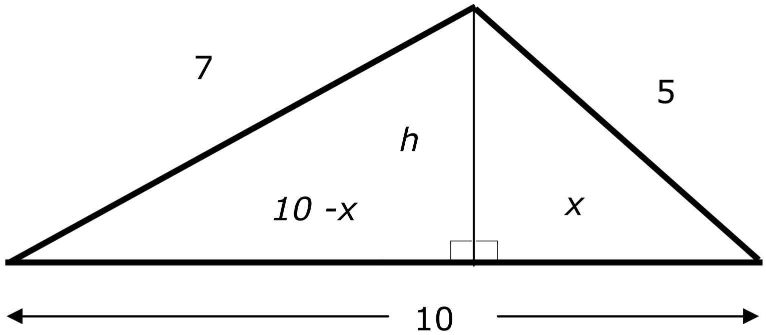


Actually, we could be a little more precise, and replace those question marks with some variables.

> Let h = the height
>
> Let x = one piece of the base.
>
> Then the other piece of the base will = 10 –x

See the following page for a diagram.

That leaves us with just two unknowns. We know from basic algebra that if we have two equations and two unknowns, we can solve for the unknowns. So we ought to be able to do this.



So now we can summarize what we know (some relationships, *en otras palabras*):

1. The two parts of the base add up to 10, so we can call one of them $10\text{-}x$ and the other x.
2. The height of the triangle, relative to the base of 10, we'll call h.
3. We know from the Pythagorean Theorem that

$$(10\text{-}x)^2 + h^2 = 7^2$$

and

$$x^2 + h^2 = 5^2$$

But what to do about all those unknowns, x and h ?

Since both equations have h^2 in them, we can get restate them as h^2 = *whatever*

$$h^2 = 7^2 - (10 - x)^2$$

and

$$h^2 = 5^2 - x^2$$

So we can get h^2 out of the picture by setting the right sides of the two equations equal to each other (since they both = h^2)

$$7^2 - (10 - x)^2 = 5^2 - x^2$$

expanding, we get

$$7^2 - (100 - 20x - x^2) = 5^2 - x^2$$

and then

$$7^2 - 100 + 20x - x^2 = 5^2 - x^2$$

Notice that we can get rid of the x^2 too! (That will be convenient!)

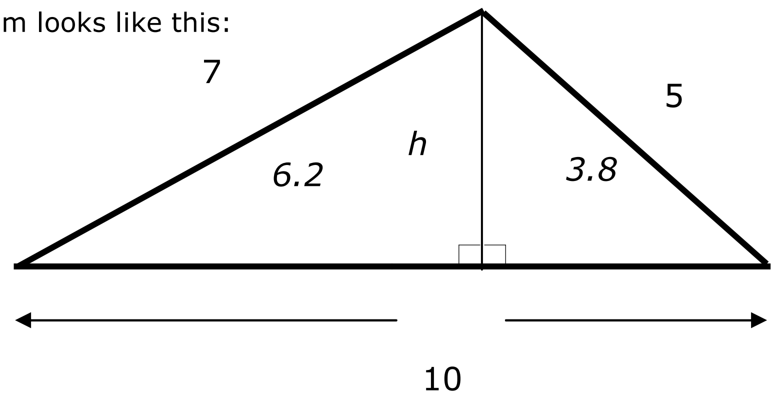
$$\begin{array}{rcl} 7^2 - 100 + 20x - x2 & = & 5^2 - x2 \\ - x^2 & = & - x^2 \\ \hline 7^2 - 100 + 20x & = & 5^2 \end{array}$$

And $\qquad 20x = 5^2 + 100 - 7^2$

So $\qquad x = \dfrac{25 + 100 - 49}{20}$

which means $\boxed{\begin{array}{rcl} x & = & 3.8 \\ (10 - x) & = & 6.2 \end{array}}$

Now our new diagram looks like this:

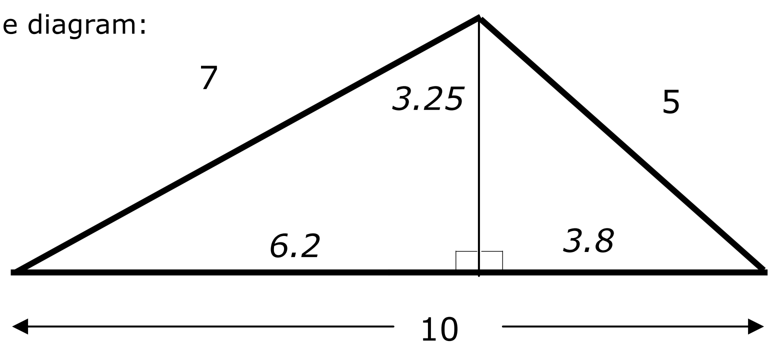


From the Pythagorean Theorem, we know that

	$(6.2)^2 + h^2 = 7^2$	*and*	$(3.8)^2 + h^2 = 5^2$
So	$h^2 = 7^2 - (6.2)^2$	*and*	$h^2 = 5^2 - (3.8)^2$
So	$h^2 = 49 - 38.44$	*and*	$h^2 = 25 - 14.44$
So	$h^2 = 10.56$	*and*	$h^2 = 10.56$
So	$h = \sqrt{10.56}$	*and*	$h = \sqrt{10.56}$

$$\boxed{\therefore\ h \approx 3.25}$$

Now we can fill in the diagram:



The area, of course, is easy:

$$A = \frac{bh}{2}$$

What is that ≈ sign doing there instead of the = sign?

$$A \approx \frac{(10)(3.25)}{2}$$

$$\boxed{A \approx 16.5 \text{ square units}}$$

The next thing would be to use variables a, b, and c instead of 7, 5, and 10.

Then we would have an actual formula that could be applied to any triangle with known sides.

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